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ALGORITHM DEVELOPMENT FOR SOLUTION OF DYNAMIC POROELASTICITY EQUATIONS BASED ON THE SPECTRAL METHOD

ABSTRACT

Dissertation by Blieva D.N. for the degree of Doctor of Philosophy (PhD) in the 6D060200-Computer Science specialty.

Research rationale. The world around us is full of examples of elastoporous fluid-saturated media, including soils, rocks, geological structures, various biological tissues of living organisms and humans, trees and plants.

Mathematical modeling of processes occurring in porous media filled with liquid is an important and relevant direction in modern research, since it allows solving important scientific and technical problems by conducting both theoretical and experimental research with the widespread use of computing technologies.

In view of the need to solve practical problems in various fields of geophysics, biomechanics and oil and gas field development, in recent decades, numerical modeling of the process of propagation of seismic waves in fluid-saturated porous media has been developed. The created realistic models of a porous medium, consisting of an elastically deformable matrix filled with a viscous fluid, make it possible to explain the observed effects during seismic studies of the properties of rocks in the presence of pore fluid.

The widespread practical application of numerical modeling of the propagation of seismic waves in a porous medium saturated with liquid to solve problems in various fields such as geophysics, biomechanics, and oil development has attracted significant attention of researchers in recent years.

An analysis of the development of methods and mathematical models shows that modeling the physical properties of a porous medium and related studies of fluid flows in porous structures traditionally occupy one of the important places among modern problems of computational mathematics and mathematical modeling. Porous media can represent the structure of various natural and artificial materials, such as soils, rocks, biological tissues, fiber, powder and foam metals, ceramics, polymers and composite materials. The need for mathematical modeling of the physical properties of such materials is dictated by the complexity of both theoretical and experimental studies of the internal structure of porous media. Without taking into account such complexity, it is impossible to predict and evaluate the effectiveness of the use of porous materials in new and modernized technological processes. The use of porous structure models has greatly influenced the development of many areas of scientific research, such as filtration and energy theories, mechanics and materials science, medicine and biology, agriculture and geosciences.

Thus, the thesis presents an algorithm for the numerical-analytical solution of the initial boundary value problem for a symmetric system of partial differential equations of t-hyperbolic type. This result has important industrial applications. In particular, seismic wave propagation simulation modeling is used for seismic exploration, as the most reliable geophysical method used to identify petroleum-bearing objects in geological structures, and is also critical for research in various fields such as biomedicine, chemical engineering, micro- and nanofluidics.

Thesis objective: The objective of this thesis work is to develop an algorithm and study methods for numerical-analytical solution of an initial boundary value problem for a system of dynamic poroelasticity equations describing the propagation of seismic waves in a fluid-saturated porous medium.

To achieve the objective the following tasks need to be completed:

- Explore existing methods for analytical and numerical solution of dynamic equations of poroelasticity, analyze existing mathematical models that describe the propagation of acoustic waves in an elastoporous medium.
- Explore a model that describes dynamic processes occurring in a single-layer porous medium characterized by physical parameters.
- Justify the accuracy of the formulation of the original initial-boundary value problem for solving dynamic equations, determine the conditions for the solvability of the problem, and obtain an analytical solution in explicit form.
- Develop an effective algorithm for a numerical solution, carry out a numerical implementation with subsequent verification for a large area of definition of spatial variables and real values of the initial physical parameters.
- Construct visualizations of the results of computational processes using an automated interface.

Subject of research. Dynamic processes occurring in a poroelastic medium saturated with liquid and its physical properties during the propagation of seismic waves therein.

Scope of research. A mathematical model for the two-dimensional case and a difference scheme that approximates it, realistically describing the physical processes occurring in a fluid-saturated elastoporous medium in a half-plane characterized by various physical parameters.

Research method: A system of linearized dynamic equations is solved for the two-dimensional problem of seismic wave propagation in porous media, written in terms of the velocity vectors of the solid matrix and the saturating fluid. To analytically solve this problem, the method of applying integral Fourier-Laplace transforms with respect to time and space variables is used. For the numerical solution, a finite-difference upwind scheme on a staggered (chessboard) grid was used, which made it possible to avoid strong oscillations of the values of grid functions at the nodes of the difference grid.

Scientific novelty of the work. An effective algorithm for the numerical solution of the dynamic poroelasticity problem has been developed in the high-performance Julia language.

Theoretical significance of research: The application of the integral Fourier-Laplace transform in time and in one of the spatial variables made it possible to reduce the original initial-boundary value problem for a symmetric-hyperbolic system of partial differential equations to solving a system of ordinary differential equations with respect to the second spatial variable. This approach made it possible to theoretically formulate a theorem on the correctness of the formulation of the problem under study, to determine the necessary and sufficient conditions for the solvability of the problem, and to obtain an explicit expression for the analytical solution.

Practical significance of the results obtained: To numerically solve a discrete analogue of the original problem, several numerical methods were used and the use of an explicit finite-difference upwind scheme on a spaced grid made it possible to obtain the most effective algorithm for parallelizing computational processes. A software package for a simulation model has been developed that describes the physical processes and properties of poroelastic media with visualization corresponding to experimental observations.

Thesis statements submitted for defense: The correctness of the formulation of the initial boundary value problem for a symmetric-hyperbolic system of partial differential equations is confirmed. Based on the constructed analytical solution, necessary and sufficient conditions for the solvability of the system of equations are identified.

- An algorithm has been developed for parallel calculations of the numerical solution of the dynamic problem of poroelasticity based on an explicit finite-difference upwind scheme on a spaced grid. Numerical solution of the problem, implemented in a software package in the modern programming language Julia (developed at MIT for high-performance computing)
- An automated complex of computer programs with an interface for visualizing the results of parallel computing processes has been developed.

Volume and structure of the work. The thesis consists of an introduction, four sections, a conclusion, three appendices with software codes and a list of used sources of 56 items; contains 112 pages of basic computer text, including 16 figures and 3 tables.

The **Introduction** provides a description of the problem and a brief overview of the subject area. The relevance of the thesis work is substantiated, a review of literary sources is carried out.

The **First section** is focused on the current state of formulation of the dynamic problem of poroelasticity based on the Dorovsky model and considers the formulation of the problem for the two-dimensional case, which is a system of eight partial differential equations with initial and boundary conditions.

The **Second section** examines the accuracy of the formulation of the original problem, the conditions for its solvability, and obtaining an analytical solution in explicit form using the Fourier-Laplace integral transform.

The **Third section** presents an algorithm for the numerical solution of the problem based on a finite-difference explicit scheme on a staggered grid.

The **Fourth section** is focused on a description of numerical experiments and an automated set of computer programs for visualizing the results obtained, which describe the physical processes occurring during the propagation of acoustic waves in a fluid-saturated porous medium.

In Conclusion, the main results obtained in the work and their significance for the field of knowledge in which they relate, as well as for the field of their practical application, are formulated.

Appendix A provides the MatLab code to obtain the analytical solution given in the second section.

Appendix B provides code in the Julia programming language for the numerical solution of the dynamic problem.

Appendix B provides the HTML code for the automated computational process with a convenient data entry interface and visualization of the calculation results displayed on a dashboard.

Confidence level and appraisal results. The validity and reliability of the study meet the requirements

The main provisions of the thesis have been appraised and presented in 15 published works, of which:

- 2 articles in international scientific journals indexed in Web of Science with a high impact factor Q1: (Computer Science – miscellaneous),

- 4 articles in journals from the list recommended by the Committee for Control in the Sphere of Education and Science of the Ministry of Education and Science of the Republic of Kazakhstan (KKSON):

- 9 publications in materials of international and national conferences and reports at scientific seminars:

- at the International Scientific and Practical Conference “Russian Science in the Modern World”, May 2018, Russian Federation, Moscow,

- Scientific conference of the Institute of Information and Computing Technologies of the Ministry of Education and Science of the Republic of Kazakhstan "Modern problems of informatics and computing technologies", Almaty, July 2-5, 2018.

- at the II Inernational Scientific and Practical Conference: “Modern world economy: problems and prospects in the era of the development of digital technologies and biotechnology” , May 15-16, 2019, Moscow

- at the I International Conference on Transfer between Mathematics & Industry (CTMI 2019), July 22-24, 2019, Santiago de Compostela, Spain.

- at the International Scientific Conference “Inverse problems in finance, economics and life sciences”. August 31 – September 4, 2019, Almaty.

- IV International Scientific and Practical Conference "Computer Science and Applied Mathematics", dedicated to the 70th anniversary of professors T.N. Biyarov, Waldemar Vuytsik and the 60th anniversary of Professor E.N. Amirgaliev, September 25-29, 2019, Almaty, Kazakhstan

- in a report at the Seminar on Differential Equations and Functional Analysis, organized by the Institute of Mathematics in collaboration with the Department of Statistics, Mathematical Analysis and Optimization of the University of Santiago de Compostela, October 24, 2019, Santiago de Compostela, Spain.

- at the meetings of the scientific seminar of the Institute of Information and Computing Technologies of the Scientific Committee of the Ministry of Education and Science of the Republic of Kazakhstan

- at a scientific seminar of the Department of Informatics of the Faculty of Information Technologies of Al-Farabi KazNU.

- at the scientific seminar of Professor Edriss Titi at the Faculty of Mathematics of Texas A&M University. Professor Edriss Titi is also Head of the Department of Mathematics at the University of Cambridge.